Math 1203 Test 2
May 1, 2019

Name: SOLUTIONS

Note that both sides of each page may have printed material.

If you could read the directions before asking me a question

Instructions:

1. Read the instructions.
2. Panic!!! Kidding, don't panic! I repeat, do NOT panic!
3. Complete all problems in the actual test. Bonus problems are, of course, optional. And they will only be counted if all other problems are attempted.
4. Show ALL your work to receive full credit. You will get 0 credit for simply writing down the answers.
5. Write neatly so that I am able to follow your sequence of steps and box your answers.
6. Read through the exam and complete the problems that are easy (for you) first!
7. Scientific calculators are allowed, but you are NOT allowed to use notes, or other aids—including, but not limited to, divine intervention/inspiration, the internet, telepathy, knowledge osmosis, the smart kid that may be sitting beside you or that friend you might be thinking of texting. In fact, **cell phones should be out of sight!**
8. Use the correct notation and write what you mean! x^2 and $x2$ are not the same thing, for example, and I will grade accordingly.
9. Other than that, have fun and good luck!

When u finally start studying hard
and Jhevon be like

NO MORE QUIZZES!!

1. Find $\frac{dy}{dx} = y'$ for the following. Simplify your answers. (5 points each)

(a) $y = \frac{2x^3 - xe^x}{3x} = \frac{2}{3}x^2 - \frac{1}{3}e^x$

$$\Rightarrow \boxed{y' = \frac{4}{3}x - \frac{1}{3}e^x}$$

(b) $y = 2\sqrt[3]{x} - \frac{3}{\sqrt{x}} + \ln(x^3+7)^2 - (\ln(x^3+7))^2$

$$= 2x^{1/3} - 3x^{-1/2} + 2\ln(x^3+7) - (\ln(x^3+7))^2$$

$$\Rightarrow \boxed{y' = \frac{2}{3}x^{-2/3} + \frac{3}{2}x^{-3/2} + \frac{6x^2}{x^3+7} - 2\ln(x^3+7)\cdot\frac{3x^2}{x^3+7}}$$

(c) $y = \frac{x^5}{x^5 - 5}$

$$\Rightarrow y' = \frac{(x^5-5)(5x^4) - x^5(5x^4)}{(x^5-5)^2}$$

$$= \frac{5x^4(x^5 - 5 - x^5)}{(x^5-5)^2}$$

$$= \boxed{\frac{-25x^4}{(x^2-5)^2}}$$

(d) $3x - 2y + x^2 + y^2 + 2xy^3 = 4 - 2x$

$$\Rightarrow 3 - 2y' + 2x + 2yy' + 2y^3 + 2x\cdot 3y^2 y' = -2$$

$$\Rightarrow y'(-2 + 2y + 6xy^2) = -2 - 3 - 2x - 2y^3$$

$$\Rightarrow \boxed{y' = \frac{-(5 + 2x + 2y^3)}{-2 + 2y + 6xy^2}}$$

2. (a) A certain product has demand function $p = 300 - 0.05x$ and cost function $C(x) = 200x + 4100$, for x items sold (the cost here is measured in dollars).

   i. (6 points) What is the revenue function, $R(x)$, and profit function, $P(x)$ for this product?

   $R(x) = xp \Rightarrow \boxed{R(x) = 300x - 0.05x^2}$

   $P(x) = R(x) - C(x) \Rightarrow \boxed{P(x) = -0.05x^2 + 100x - 4100}$

   ii. (2 points) Find the marginal cost and marginal revenue functions.

   Marginal cost $= \boxed{C'(x) = 200}$

   Marginal Revenue $= \boxed{R'(x) = 300 - 0.1x}$

   iii. (2 points) Assume 100 units of the product is made, using the appropriate marginal function, estimate the additional revenue gained from producing and selling the 101st unit.

   $$\text{Additional revenue} \approx R'(100) = 300 - 0.1(100) = \boxed{290}$$

3. The half-life of cesium-137 is 30 years. Suppose we start with a 100-mg sample, and let $P(t)$ represent the amount of cesium remaining after time t.

   (a) (4 points) Find and simplify $P(t)$

   $\text{Half-life} = \frac{\ln 2}{r}$

   $\Rightarrow r = \frac{\ln 2}{30} \qquad \Rightarrow \boxed{P = 100e^{-\frac{\ln 2}{30}t}}$

   Also, $P_0 = 100$

   (b) (3 points) How much of the sample will remain after 100 years?

   $$P(100) = \boxed{100e^{-\frac{\ln 2}{30}(100)}}$$

   (c) (3 points) After how long will only 1 mg remain?

   Set $1 = 100e^{-\ln 2/30\, t}$

   $\Rightarrow \frac{1}{100} = e^{-\ln 2/30\, t}$

   $\Rightarrow \ln\frac{1}{100} = -\frac{\ln 2}{30}t$

   $\Rightarrow \boxed{t = -\frac{30\ln\frac{1}{100}}{\ln 2}}$

   OR

   $\boxed{t = \frac{30\ln 100}{\ln 2}}$

4. (10 points) The volume V of a cancer tumor is given by $V = \pi x^3/6$ where x is the radius of the tumor. Using math he learned from Jhevon, a doctor estimates that the radius is growing at a rate of 0.4 mm per day at a point when the radius is 10 mm. How fast is the volume of the tumor changing at this time?

It's naht a toomah!!!

① Read!

② Diagram

Skip!

③ Know | Want

$\frac{dx}{dt} = 0.4$ | $\frac{dV}{dt}$ when $x = 10$

④ Equation

Given: $V = \frac{\pi x^3}{6}$

⑤ Differentiate

$$\frac{dV}{dt} = \frac{\cancel{3}\pi x^2}{\cancel{6}_{2}} \frac{dx}{dt}$$

⑥ Plug In

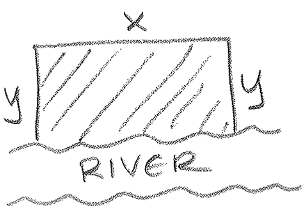
$$\frac{dV}{dt} = \frac{\pi(10)^2}{2}(0.4)$$

$$= \boxed{20\pi} \ mm^3/day$$

5. (20 points) A farmer has 2400 ft of fencing to fence around a rectangular field that is alongside a straight river. The river will form the border of one of the sides, and so he needs no fencing along the river. Use calculus to assist you in finding the dimensions of the field he can fence around that has maximum area.

① Read!

② Diagram



③ Constraint

Fencing

$\Rightarrow x + 2y = 2400$

$\Rightarrow x = 2400 - 2y$

Objective

Area

$A = xy$

④ Plug constraint into objective

$A = (2400 - 2y)y$

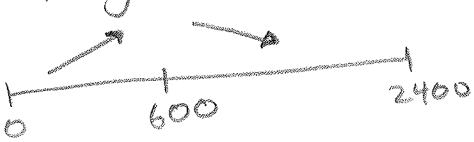
$= 2400y - 2y^2$

⑤ Maximize A (w/ Calculus)

Set $A' = 0$ or ~~und~~ → never.

$\Rightarrow 2400 - 4y = 0$

$\Rightarrow y = 600$



$\Rightarrow x = 2400 - 2(600)$

$= 1200.$

⑥ Answer the question!

Dimensions:

$\boxed{600 \times 1200}$ ft

(where the side parallel to the river is 1200 ft.)

6. (20 points) Sketch the graph of the function $f(x) = x^3 - 3x^2 + 4 = (x+1)(x-2)^2$ by first finding (provided they exist) the domain, intercepts, asymptotes, local extrema, inflection point(s), intervals of increasing and decreasing, and intervals of concavity. The preceding must be indicated on your graph.

① Domain: $(-\infty, \infty)$ (1pt)

② Intercepts: (2pts)

x-int: $(x+1)(x-2)^2 = 0$

$\Rightarrow \boxed{x=-1,\ x=2}$

y-int: $\boxed{y=4}$

③ Asymptotes (1pt)

None! (Polynomial)

④ Inc/Dec/Max/Min (6pts)

$f' = 3x^2 - 6x$

crit pt: $3x^2 - 6x = 0$

$\Rightarrow 3x(x-2) = 0$

$\Rightarrow x = 0,\ x = 2$

(+) (−) (+) f'

-1 0 1 2 3

$\Rightarrow$ Inc: $(-\infty, 0) \cup (2, \infty)$

Dec: $(0, 2)$

Max: $(0, 4)$

Min: $(2, 0)$

⑤ C.U./C.D./Inf (5pts)

$f'' = 6x - 6$

crit pts: $6x - 6 = 0$

$\Rightarrow x = 1$

(−) (+) f''

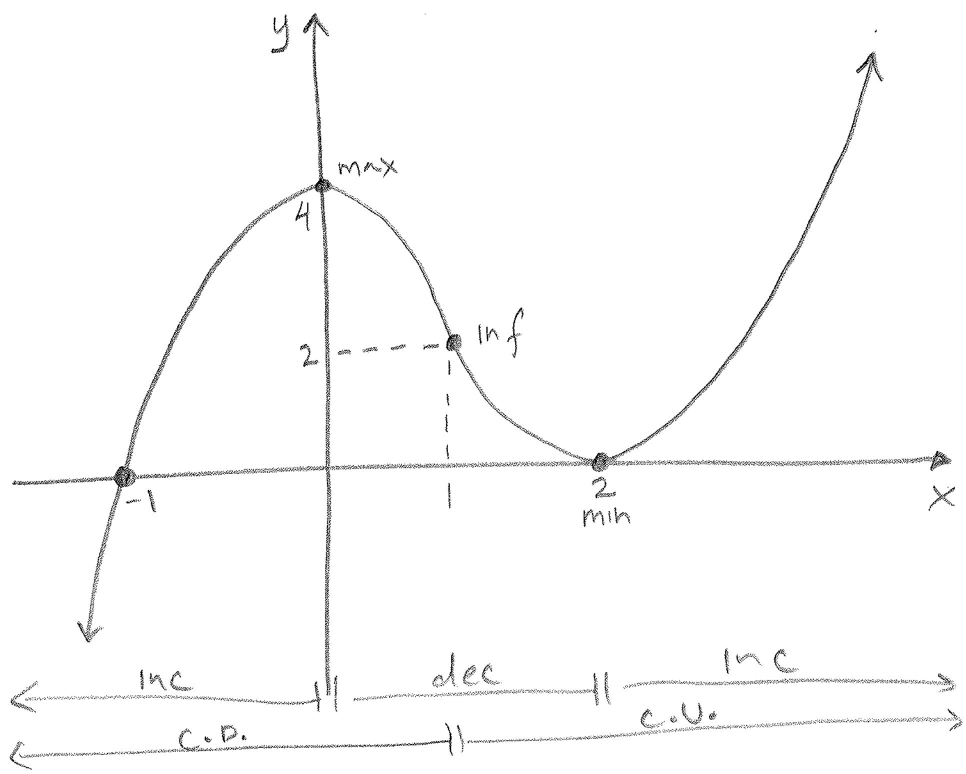
0 1 2

$\Rightarrow$ C.U.: $(1, \infty)$

C.D.: $(-\infty, 1)$

Inf: $(1, 2)$

⑥ Sketch (5pts)



7. (10 points) Let $f(x) = x^3 - 3x^2 + 4$, find the absolute maximum and minimum of $f(x)$ on the interval $[1,4]$.

① Crit Pts (6 pts)

for crit. pts

$f' = 3x^2 - 6x = 0$ or ~~undef~~

$\Rightarrow 3x(x-2) = 0$

$\Rightarrow \cancel{x=0},\ x = 2$

reject! Not in interval

$f(2) = 0$

(3 pts)

② Endpoints

$f(1) = 2$

$f(4) = 20$

(1 pt)

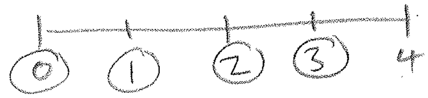
③ Compare and conclude

$$\boxed{\begin{aligned} f(4) &= 20 \rightarrow \text{abs max} \\ f(2) &= 0 \rightarrow \text{abs min} \end{aligned}}$$

Bonus Problems: (You must complete all problems in the actual test to be eligible).

1. (8 points) Using Riemann sums with 4 subintervals, approximate the area under $f(x) = 16 - x^2$ on the interval $[0,4]$ using left hand endpoints. Show all your work.

$$\Delta x = \frac{b-a}{n} = \frac{4-0}{4} = 1$$

$\Rightarrow$ 

$$\Rightarrow A \approx L_4 = (f(0) + f(1) + f(2) + f(3))(\Delta x)$$
$$= (16-0^2 + 16-1^2 + 16-2^2 + 16-3^2)(1)$$
$$= \boxed{50}$$

2. (8 points) Find the exact area under $f(x) = 16 - x^2$ on $[0,4]$. Is your approximation in problem 1 an over or underestimate?

$$A = \int_0^4 16 - x^2\, dx$$
$$= 16x - \frac{x^3}{3}\Big|_0^4$$
$$= 64 - \frac{64}{3}$$
$$= \boxed{\frac{128}{3}}$$

$$\approx 42.666\ldots$$

$\Rightarrow$ Approximation in problem 1 was an $\boxed{\text{overestimate}}$

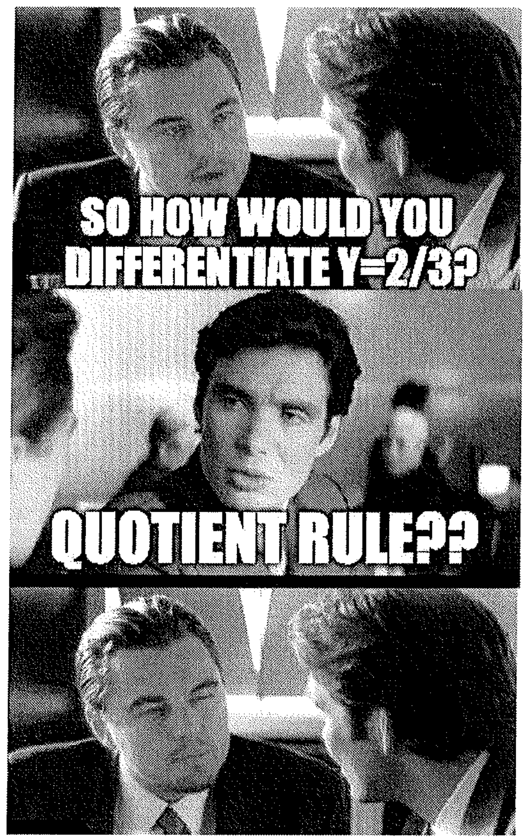
3. (1 point each, all or nothing!) Complete the following formulas.

(a) $\int x^n dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1$

(b) $\int 1/x\ dx = \ln|x| + C,\ x \neq 0$

(c) $\int e^{kx}\, dx = \frac{1}{k} e^{kx} + C,\ k \neq 0$

(d) $\int a^x\, dx = \frac{a^x}{\ln a} + C,\ a > 1 \text{ or } 0 < a < 1$


SO HOW WOULD YOU
DIFFERENTIATE Y=2/3?
QUOTIENT RULE??