Math 1203 Review for Test 2

April 21, 2019

Name: SOLUTIONS

Note that both sides of each page may have printed material.

Instructions:

1. Read the instructions.

2. Panic!!! Kidding, don't panic! I repeat, do NOT panic!

3. Complete all problems in the actual test. Bonus problems are, of course, optional. And they will only be counted if all other problems are attempted.

4. Show ALL your work to receive full credit. You will get 0 credit for simply writing down the answers.

5. Write neatly so that I am able to follow your sequence of steps and box your answers.

6. Read through the exam and complete the problems that are easy (for you) first!

7. Calculators are NOT allowed. Also, you are NOT allowed to use notes, or other aids—including, but not limited to, divine intervention/inspiration, the internet, telepathy, knowledge osmosis, the smart kid that may be sitting beside you or that friend you might be thinking of texting.

8. In fact, **cell phones should be out of sight!**

9. Use the correct notation and write what you mean! x^2 and $x2$ are not the same thing, for example, and I will grade accordingly.

10. Other than that, have fun and good luck!

1. Find $\frac{dy}{dx} = y'$ for the following. Simplify your answers. (5 points each)

(a) $y = \frac{5x^3+3x^2}{2x}$ $= \frac{5}{2}x^2 + \frac{3}{2}x$

$\Rightarrow \boxed{y' = 5x + \frac{3}{2}}$

(b) $y = 2\sqrt{x} + \frac{5}{\sqrt[3]{x}} - \ln(x^2+1)^3 + e^{x^2}$

$= 2x^{1/2} + 5x^{-1/3} - 3\ln(x^2+1) + e^{x^2}$

$\Rightarrow \boxed{y' = x^{-1/2} - \frac{5}{3}x^{-4/3} - \frac{6x}{x^2+1} + 2xe^{x^2}}$

(c) $y = \frac{x^6}{4+x^6}$

$y' = \frac{(4+x^6)(6x^5) - x^6(6x^5)}{(4+x^6)^2}$

$= \frac{6x^5(4+x^6-x^6)}{(4+x^6)^2}$

$\Rightarrow \boxed{y' = \frac{24x^5}{(4+x^6)^2}}$

(d) $x^2y^3 + 2x + 3y = 5x + 12$

$\Rightarrow 2xy^3 + x^2 \cdot 3y^2y' + 2 + 3y' = 5$

$\Rightarrow y'(3x^2y^2+3) = 5 - 2xy^3 - 2$

$\Rightarrow \boxed{y' = \frac{3-2xy^3}{3+3x^2y^2}}$

2. We return to our story, where our hero, Jhevon, is trying to get his hotdog business to be the very best, like no one ever was. The cost $C(x)$, in dollars, of producing x hotdogs is given by

$$C(x) = 50 - 20x + 2x^2$$

Assuming Jhevon will sell only specialty hotdogs at \$5/hotdog, answer the following:

i. (5 points) What is Jhevon's revenue function, $R(x)$, and profit function, $P(x)$?

$\boxed{R(x) = 5x}$, $P(x) = R(x) - C(x) \Rightarrow \boxed{P(x) = -2x^2 + 25x - 50}$

ii. (5 points) Find the marginal cost and marginal revenue functions.

$\boxed{C'(x) = -20 + 4x}$ $\boxed{R'(x) = 5}$

iii. (5 points) Assume Jhevon made 6 hotdogs, use the marginal cost to approximate how much more it would cost him to make the seventh.

$C'(6) = -20 + 4(6) = 4$

$\boxed{\text{It would cost \$4 to make the } 7^{th}}$

3. (5 points each part) A population $P(t)$ doubles every 3 years. In 1990, the population was 4 million. Assume exponential growth and that we start measuring the population in 1990.

(a) Find and simplify $P(t)$

$t_d = \frac{\ln 2}{r}$

$\Rightarrow r = \frac{\ln 2}{3}$ $\Rightarrow$ $\boxed{P(t) = 4e^{\frac{\ln 2}{3}t}, \text{ in millions}}$

(b) What is the population's size in 1992?

$\boxed{P(2) = 4e^{\frac{2\ln 2}{3}} \text{ million}}$

(c) After how long will the population be 10 million?

Set $10 = 4e^{\frac{\ln 2}{3}t}$

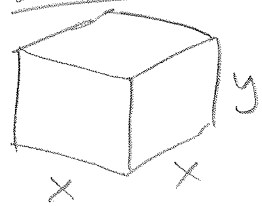
$\Rightarrow \frac{5}{2} = e^{\frac{\ln 2}{3}t}$

$\Rightarrow \ln\left(\frac{5}{2}\right) = \frac{\ln 2}{3}t$

$\Rightarrow \boxed{t = \frac{3\ln\left(\frac{5}{2}\right)}{\ln 2}}$ years

4. (20 points) A closed rectangular box with a square base and volume 12 cubic feet is to be constructed using two different types of materials. The top is made of metal costing \$2 per square foot, and the remaining sides and base is made of wood costing \$1 per square foot. Find the dimensions of the box for which the cost of construction is minimized.

Diagram



Constraint

$$\text{Vol} = 12 = x^2 y$$

$$\Rightarrow y = \frac{12}{x^2}$$

Objective

$$C = \underbrace{2\cdot x^2}_{\text{top}} + \underbrace{1\cdot 4xy}_{\text{sides}} + \underbrace{1\cdot x^2}_{\text{base}}$$

$$\Rightarrow C = 3x^2 + 4xy$$

New objective

$$C = 3x^2 + 4x\left(\frac{12}{x^2}\right)$$

$$\Rightarrow C = 3x^2 + \frac{48}{x}$$

Minimize

for min

$$C' = 6x - \frac{48}{x^2} = 0 \text{ or } \cancel{\text{und}}$$

$$\Rightarrow 6x^3 - 48 = 0$$

$$\Rightarrow x^3 = \frac{48}{6} = 8$$

$$\Rightarrow x = 2$$

$$\Rightarrow y = \frac{12}{2^2} = 3$$

$$\Rightarrow \boxed{\text{Dimensions are: } 2 \times 2 \times 3} \text{ ft}$$

5. (10 points) Sketch the graph of the function $f(x) = x^3 - 12x^2 + 36x$ by first finding (provided they exist) the domain, intercepts, asymptotes, local extrema, inflection point(s), intervals of increasing and decreasing, and intervals of concavity. The preceding must be indicated on your graph.

Domain

$\boxed{(-\infty, \infty)}$

Intercepts

x-Int: $x^3 - 12x^2 + 36x = 0$

$x(x^2 - 12x + 36) = 0$

$x(x-6)(x-6) = 0$

$\boxed{x = 0,\ x = 6} \to$ x-Int

y-Int: $\boxed{y = 0} \to$ y-Int

Asymptotes

None (polynomial).

Inc/Dec/Max/Min

$f' = 3x^2 - 24x + 36$

$= 3(x^2 - 8x + 12)$

$= 3(x-6)(x-2)$

$x = 2,\ x = 6 \to$ crit pts

f': (+) 0 | 2 max | (−) 3 | 6 min | (+) 7

$f(2) = 32 \to (2, 32)$ max

$f(6) = 0 \to (6, 0)$ min

Inc: $(-\infty, 2) \cup (6, \infty)$
Dec: $(2, 6)$
Max: $(2, 32)$
Min: $(6, 0)$

C.U / C.D / Inf

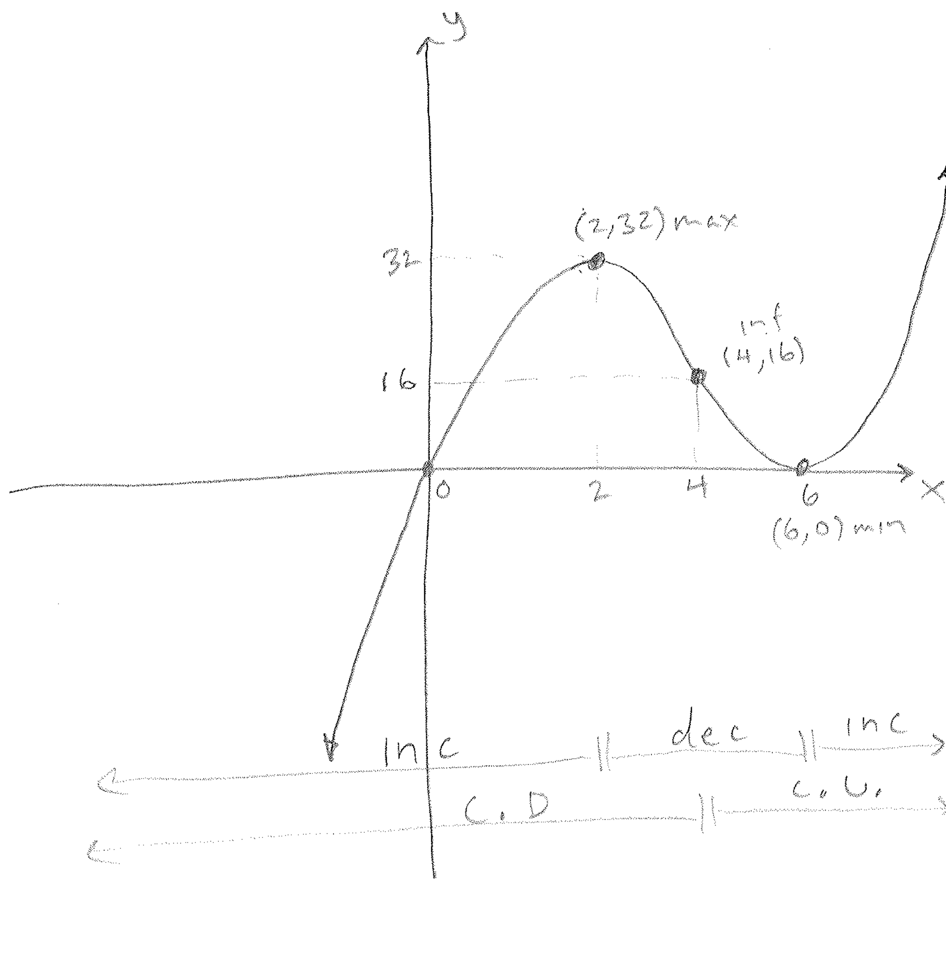
$f'' = 6x - 24$

crit pts: $6x - 24 = 0$
$\Rightarrow x = 4$

f'': (−) 0 | 4 Inf | (+) 5

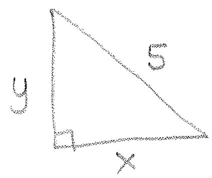
$f(4) = 16 \to$ Inf

C.U: $(4, \infty)$
C.D: $(-\infty, 4)$
Inf: $(4, 16)$



6. (10 points) A 5 foot ladder leans against a vertical wall. Batman pushes the foot of the ladder towards the wall at a rate of 2 ft/sec. At what rate is top of the ladder moving along the wall when the foot of the ladder is 3 feet from the wall? Include a sketch in your answer.

Diagram



Know

$\frac{dx}{dt} = -2$

Want

$\frac{dy}{dt}$ when $x = 3$

Equation

$x^2 + y^2 = 25$

NB, when $x=3, y=4$ ($3-4-5\ \Delta$)

Differentiate

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$\Rightarrow \frac{dy}{dt} = \frac{-x\frac{dx}{dt}}{y}$

$= \frac{-3(-2)}{4}$

$= \boxed{\frac{3}{2}}$ ft/s

7. (10 points) Let $f(x) = 2x^3 - 3x^2 - 12x + 1$, find the absolute maximum and minimum of $f(x)$ on the interval $[-2,1]$.

1/ Crit pts

$f' = 6x^2 - 6x - 12 \overset{\text{set}}{=} 0$

$\Rightarrow 6(x^2 - x - 2) = 0$

$\Rightarrow 6(x-2)(x+1) = 0$

$\Rightarrow x = 2,\ x = -1$

$f(-1) = 8$

$f(2)$ not in interval

2/ End pts

$f(-2) = -3$

$f(1) = -12$

3/ Compare

$$\boxed{\begin{array}{l} f(-1) = 8 \longrightarrow \text{abs max} \\ f(1) = -12 \longrightarrow \text{abs min} \end{array}}$$

Bonus Problems: (You must complete all problems in the actual test to be eligible).

1. (5 points) Using Riemann sums with 4 subintervals, approximate the area under $f(x) = x^2 + 1$ on the interval $[-1,3]$ using left hand endpoints. Show all your work.

$-1 \quad 0 \quad 1 \quad 2 \quad 3$

$\Delta x = \frac{3-(-1)}{4} = 1$

$$\Rightarrow A \approx (f(-1) + f(0) + f(1) + f(2))(1)$$
$$= [(-1)^2+1 + 0^2+1 + 1^2+1 + 2^2+1]$$
$$= \boxed{10}$$

2. (5 points) Find the exact area under $f(x) = x^2 + 1$ on $[-1,3]$. Is your approximation in problem 1 an over or underestimate?

$$A = \int_{-1}^{3} x^2+1\, dx = \frac{x^3}{3} + x \Big|_{-1}^{3}$$
$$= 9 + 3 - \left(-\frac{1}{3} - 1\right)$$
$$= 12 + \frac{4}{3}$$
$$= \boxed{\frac{40}{3}} \rightarrow \boxed{\text{Our approx was an underestimate!}}$$

3. (5 points) The half-life of a radioactive substance is 1200 years. Find and simplify $P(t)$, the amount of substance remaining after t years.

$$\Rightarrow 1200 = \frac{\ln 2}{r}$$
$$\Rightarrow r = \frac{\ln 2}{1200}$$
$$\Rightarrow \boxed{P(t) = P_0 e^{-\frac{\ln 2}{1200} t}}, \quad P_0 = \text{initial amount.}$$

4. (5 points) Complete the following formulas.

(a) $\int x^n dx = \boxed{\frac{x^{n+1}}{n+1} + C}, \quad n \neq -1$

(b) $\int 1/x\ dx = \boxed{\ln|x| + C}, \quad x \neq 0$

(c) $\int e^{kx}\, dx = \boxed{\frac{1}{k} e^{kx} + C} \quad k \neq 0$

(e) $\int a^x\, dx = \boxed{\frac{a^x}{\ln a} + C}, \quad a > 1,\ 0 < a < 1$