Math 1203 Quiz 10

Day after April Fool's Day 2019

Name: ANSWERS

Instructions: No calculators. Use provided scrap. Write your fully simplified answers in the space provided.

1. Use linear approximation or differentials to approximate $(27.1)^{1/3}$ by completing the following:

   (a) Define a function to use: $f(x) =$ $\sqrt[3]{x}$ or $x^{1/3}$

   (b) $x =$ 27.1, $a =$ 27

   (c) The general formula (in f) used to make the approximation $f(x) \approx f(a) + f'(a)(x-a)$

   (d) The approximate value is $3 + \frac{1}{270} = \frac{811}{270}$ (write as a fraction)

2. A mythical bank pays 4% interest compounded continuously. Suppose you deposit \$5000 into an account with this bank. Let $P(t)$ be your account balance t years after your initial deposit.

   (a) Describe the growth of this account with a differential equation: $P' = 0.04P$

   (b) Find a formula for $P(t) =$ $5000e^{0.04t}$

   (c) How long will it take for your principal to reach \$7000? $\frac{\ln(7/5)}{0.04}$ years

   (d) At what rate (in \$/year) is your account increasing when the balance is \$7000? 280 \$/year ($0.04(7000)$)

3. Suppose $C(x) = 2x^2 + 3x$ is the cost to manufacture x units of a product.

   (a) Find the marginal cost (express as an equation): $C'(x) = 4x + 3$

   (b) What is the marginal cost when $x = 10$? 43

   (c) Interpret your answer to part (b) approximate additional cost of the 11th unit.

Bonus (Complete the other problems to be eligible):

1. Suppose $p = \frac{25}{\sqrt{x}}$ is the demand function for a product.

   (a) What is the revenue function for this product? $R(x) =$ $25\sqrt{x}$

   (b) What is the marginal revenue for this product: $R'(x) = 25/2\sqrt{x}$ (equation!)

   (c) Suppose the cost to produce x units of this product is $C(x) = x^2 + 2x$, what is the marginal profit?

   $P'(x) = 25/2\sqrt{x} - 2x - 2$ (equation!)

   (d) Is it worth producing one more item after 100 have been produced? Answer "yes" or "no" and justify:

   No! $P'(100) = -200.75 < 0$. Since marginal profit is negative, we'd expect to lose money on the next unit.