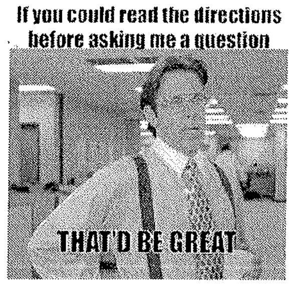
Math 1203 Test 1
March 1, 2019

Name: Solutions

Note that both sides of each page may have printed material.



Instructions:

1. Read the instructions.
2. Panic!!! Kidding, don't panic! I repeat, **do NOT panic!** Don't look down, while you're at it.
3. Complete all problems in the actual test. Bonus problems are, of course, optional. And they will only be counted if all other problems are attempted.
4. Show ALL your work to receive full credit. You will get 0 credit for simply writing down the answers.
5. Write neatly so that I am able to follow your sequence of steps and box your answers.
6. Read through the exam and complete the problems that are easy (for you) first!
7. Scientific calculators are allowed, but you are NOT allowed to use notes, graphing calculators, or other aids—including, but not limited to, divine intervention/inspiration, the internet, telepathy, knowledge osmosis, the smart kid that may be sitting beside you or that friend you might be thinking of texting. In fact, **cell phones should be out of sight!**
8. If you are caught with a cell phone, you will be asked to leave the exam and you'll be given a zero.
9. Use the correct notation and write what you mean! x^2 and $x2$ are not the same thing, for example, and I will grade accordingly.
10. Other than that, have fun and good luck!

Studied the review test? It probably won't matter...

I'm kidding! Studying is good!

1. (4 points each) Simplify the following:

(a) $\ln\left(\frac{5e^{2x}}{\sqrt{x^2-1}}\right)$ $= \ln 5 + \ln e^{2x} - \ln(x^2-1)^{1/2}$

$= \ln 5 + 2x - \frac{1}{2}\ln((x-1)(x+1))$

$= \boxed{\ln 5 + 2x - \frac{1}{2}\ln(x-1) - \frac{1}{2}\ln(x+1)}$

(b) $\frac{4(2x+1)(3x-1)^3-9(2x+1)^2(3x-1)^2}{(3x-1)^6}$

$= \frac{(2x+1)\cancel{(3x-1)^2}\left[4(3x-1)-9(2x+1)\right]}{(3x-1)^{\cancel{6}\,4}}$

$= \frac{(2x+1)(12x-4-18x-9)}{(3x-1)^4}$

$= \boxed{\frac{(2x+1)(-6x-13)}{(3x-1)^4}}$ or $\boxed{\frac{-(2x+1)(6x+13)}{(3x-1)^4}}$

(c) $e^{2\ln y-3\ln(2x)}$

$= e^{\ln y^2 - \ln(2x)^3}$

$= e^{\ln y^2} \cdot e^{-\ln(2x)^3}$

$= y^2 \cdot (2x)^{-3}$

$= \boxed{\frac{y^2}{8x^3}}$

2. (8 points) Find the average rate of change of $f(x) = 2 - x^2$ on [1,2].

$$f_{avg} = \frac{f(2) - f(1)}{2 - 1}$$

$$= \frac{2 - 2^2 - (2 - 1^2)}{1}$$

$$= -4 + 1$$

$$= \boxed{-3}$$

3. (6 points each) Let $f(x) = 3x^2 + 5$ and $g(x) = \sqrt{x^2 - 1}$.

(i) Find and simplify $f \circ g$

$$= 3(g)^2 + 5$$

$$= 3(x^2 - 1) + 5$$

$$= 3x^2 - 3 + 5$$

$$= \boxed{3x^2 + 2}$$

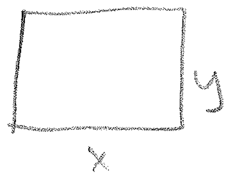
(ii) Find the domain of $f \circ g$: $\mathrm{dom}(3x^2+2) = (-\infty, \infty)$, but,

$\mathrm{dom}(g) = (-\infty, -1] \cup [1, \infty)$, since

that is where $x^2 - 1 \geq 0$.

$$\Rightarrow \boxed{\mathrm{dom}(f \circ g) = (-\infty, -1] \cup [1, \infty)}$$

4. (8 points) A farmer has 300 feet of fencing and wants to make a rectangular enclosure. What should the dimensions of the fence be in order to maximize the area the fencing encloses? Hint: Draw and label a diagram of the enclosure. Find a function that describes the area in terms of the length of one of the sides of the enclosure. Find the length that makes the area as large as possible and use this to answer the question.



We know the perimeter = 300

$\Rightarrow 2x + 2y = 300$

$\Rightarrow x + y = 150$

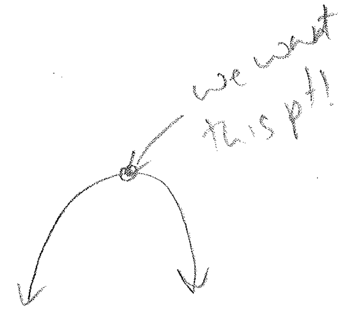
$\Rightarrow y = 150 - x$

Now, if A = area,

$A = xy$

$= x(150 - x)$

$= 150x - x^2 \rightarrow$



For max area, we need the vertex.

$\Rightarrow x = \frac{-150}{2(-1)} = 75$

$\Rightarrow y = 150 - 75 = 75.$

$\boxed{\text{Dimensions: } 75 \text{ ft} \times 75 \text{ ft}}$ for max Area

5. (10 points each) Solve the following equations:

(i) $e^{2x+3} - 7 = 0$

$\Rightarrow e^{2x+3} = 7$

$\Rightarrow \ln e^{2x+3} = \ln 7$

$\Rightarrow 2x + 3 = \ln 7$

$\Rightarrow 2x = \ln 7 - 3$

$\Rightarrow \boxed{x = \frac{\ln(7) - 3}{2}}$

(ii) $\ln(5 - 2x) = -3$

e e

$\Rightarrow 5 - 2x = e^{-3}$

$\Rightarrow 5 - e^{-3} = 2x$

$\Rightarrow \frac{5 - e^{-3}}{2} = x$

$\Rightarrow \boxed{x = \frac{5 - e^{-3}}{2}}$

6. (a) (15 points) Let $f(x) = 3x - \frac{2}{x}$. Use the limit definition of the derivative to find $f'(x)$. **No credit will be given for any other method!**

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{3(x+h) - \frac{2}{x+h} - \left(3x - \frac{2}{x}\right)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{3x} + 3h - \frac{2}{x+h} - \cancel{3x} + \frac{2}{x}}{h}$$

$$= \lim_{h \to 0} \left(\frac{3\cancel{h}}{\cancel{h}} + \frac{\frac{2}{x} - \frac{2}{x+h}}{h} \right)$$

$$= \lim_{h \to 0} \left(3 + \frac{\frac{2}{x} - \frac{2}{x+h}}{h} \cdot \frac{x(x+h)}{x(x+h)} \right)$$

$$= \lim_{h \to 0} \left(3 + \frac{2(\cancel{x}+h) - \cancel{2x}}{hx(x+h)} \right)$$

$$= \lim_{h \to 0} \left(3 + \frac{2\cancel{h}}{\cancel{h}x(x+h)} \right)$$

$$= \boxed{3 + \frac{2}{x^2}}$$

(b) (5 points) Using your answer to part (a), compute the equation of the tangent line to $f(x)$ at the point where $x = 1$. **Write your line in $y = mx + b$ form.**

$x_1 = 1 \Rightarrow y_1 = 3(1) - \frac{2}{1} = 1$

$\Rightarrow (x_1, y_1) = (1, 1)$

$m = f'(1)$

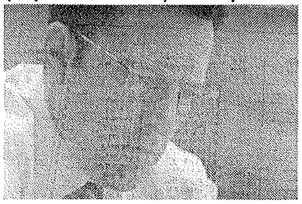
$\Rightarrow m = 3 + \frac{2}{(1)^2} = 5$

$\Rightarrow$ Using $y - y_1 = m(x - x_1)$

$\rightarrow y - 1 = 5(x - 1)$

$\Rightarrow \boxed{y = 5x - 4}$

7. (5 points each) Compute the following limits:



Maximum concentration!!!

(i) $\lim\limits_{x\to -1} \dfrac{x^2+3x+2}{x^2+1}$ plug in!

$= \dfrac{1-3+2}{2}$

$= \boxed{0}$

(ii) $\lim\limits_{x\to -\infty} \dfrac{3x^3+3}{5x^2-4}$

Top heavy

$= \lim\limits_{x\to -\infty} \dfrac{3x^3}{5x^2} \to \dfrac{(-)}{(+)}$

$= \boxed{-\infty}$

(iii) $\lim\limits_{x\to -1^-} \dfrac{x^2-4}{x^2-x-2}$

$= \lim\limits_{x\to -1^-} \dfrac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(x+1)}$

$= \lim\limits_{x\to -1^-} \dfrac{x+2}{x+1}$

$= \dfrac{1}{0^-}$

$= \boxed{-\infty}$

(ii) $\lim\limits_{x\to -\infty} \dfrac{3-2x-3x^3}{2+x+9x^3}$

$= \dfrac{-3}{9}$

$= \boxed{-\dfrac{1}{3}}$

Ratio of leading coeff.

Bonus Problems (these will only be counted if all other problems are attempted):

1. (2 points each) State the following rules precisely:

   (a) The power rule for derivatives: $\frac{d}{dx}x^n = nx^{n-1}$

   (b) The chain rule: $\frac{d}{dx}f(g(x)) = f'(g(x))\cdot g'(x)$

   (c) The quotient rule: $\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{f'g - fg'}{g^2}$

   (d) The product rule: $\frac{d}{dx}(f\cdot g) = f'g + fg'$

   (f) The rule to differentiate a general exponential with base a and power u: $\frac{d}{dx}a^u = u'a^u\ln a$

   (g) The rule to differentiate the natural logarithm of a function u: $\frac{d}{dx}\ln u = \frac{u'}{u}$

2. Find $\frac{dy}{dx} = y'$ for the following. Simplify your answers. (3 points each)

   (a) $y = \frac{2x^3 - xe^x}{3x}$

   $= \frac{2}{3}x^2 - \frac{1}{3}e^x$

   $\Rightarrow \boxed{y' = \frac{4}{3}x - \frac{1}{3}e^x}$

   (b) $y = 2\sqrt[3]{x} - \frac{3}{\sqrt{x}} - \ln(x^3+7)^3$

   $= 2x^{1/3} - 3x^{-1/2} - 3\ln(x^3+7)$

   $\Rightarrow y' = \frac{2}{3}x^{-2/3} + \frac{3}{2}x^{-3/2} - 3\cdot\frac{3x^2}{x^3+7}$

   $\Rightarrow \boxed{y' = \frac{2}{3}x^{-2/3} + \frac{3}{2}x^{-3/2} - \frac{9x^2}{x^3+7}}$

   (c) $y = \frac{x^5}{x^5-5}$

   $\Rightarrow y' = \frac{5x^4(x^5-5) - x^5\cdot 5x^4}{(x^5-5)^2}$

   $= \frac{5x^4(x^5 - 5 - x^5)}{(x^5-5)^2}$

   $\Rightarrow \boxed{y' = \frac{-25x^4}{(x^5-5)^2}}$

   (d) $y = e^{x^3} + x^x$

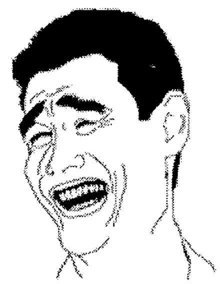
   $= e^{x^3} + e^{\ln x^x}$

   $= e^{x^3} + e^{x\ln x}$

   $\Rightarrow y' = 3x^2e^{x^3} + (\ln x + 1)e^{x\ln x}$

   $\Rightarrow \boxed{y' = 3x^2e^{x^3} + (\ln x + 1)x^x}$

JHEVON:



So how'd you do
on the test??

ME: