

Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided! Do your rough work on scrap paper.

1. Define  $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$  (using limits)

2. Let  $f(x) = 2 - x^2$ .

(a) Use the limit definition of the derivative to find  $f'(x)$ . Show your work!

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{2 - (x+h)^2 - (2 - x^2)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\cancel{2} - \cancel{x^2} - 2xh - h^2 - \cancel{2} + \cancel{x^2}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\cancel{h}(-2x - h)}{\cancel{h}} \\ &= \lim_{h \rightarrow 0} (-2x - h) \\ &= \boxed{-2x} \end{aligned}$$

3 points

- 1 pt for set up
- 1 pt for correct notation and steps (including writing "lim" every time  $h \rightarrow 0$  until limit is taken).
- 1 pt for final answer.

(b) Find the equation of the tangent line to  $f(x)$  at the point where  $x = 1$ .

$$\begin{aligned} y - 1 &= -2(x - 1) \\ \text{OR} \\ y &= -2x + 3 \end{aligned}$$

3. Complete the table:

Function (assume all are continuous everywhere)	The behavior it tells us about	How?
$f(x)$	Points on the graph	Plug in $x$ into $f(x)$ , find corresponding $y$ -value to get $(x, y)$
$f'(x)$	The function is increasing	$f'(x) > 0$
$f''(x)$	The function is Concave down	$f''(x) < 0$
$f'(x)$	The function is decreasing	$f'(x) < 0$

**Bonus:**

1. Complete the following rules:

(a)  $\frac{d}{dx}(f(x) \cdot g(x)) = f' \cdot g + f \cdot g'$  (b)  $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f' \cdot g - f \cdot g'}{g^2}$

(c)  $\frac{d}{dx}f(g(x)) = f'(g(x)) \cdot g'(x)$  (d)  $\frac{d}{dx}(f(x) \pm g(x)) = f'(x) \pm g'(x)$