Math 1203 Review for Test 1

February 23, 2019

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Note that both sides of each page may have printed material.

Instructions:

1. Read the instructions.

2. Panic!!! Kidding, don't panic! I repeat, **do NOT panic**! Don't look down, while you're at it.

3. Complete all problems in the actual test. Bonus problems are, of course, optional. And they will only be counted if all other problems are attempted.

4. **You have 1 hour and 15 minutes to complete the test.**

5. Show ALL your work to receive full credit. You will get 0 credit for simply writing down the answers.

6. Write neatly so that I am able to follow your sequence of steps and box your answers.

7. Read through the exam and complete the problems that are easy (for you) first!

8. Scientific calculators are allowed, but you are NOT allowed to use notes, graphing calculators, or other aids—including, but not limited to, divine intervention/inspiration, the internet, telepathy, knowledge osmosis, the smart kid that may be sitting beside you or that friend you might be thinking of texting.

9. In fact, **cell phones should be out of sight!**

10. Use the correct notation and write what you mean! x^2 and $x2$ are not the same thing, for example, and I will grade accordingly.

11. Other than that, have fun and good luck!

    May the force be with you. But you can't ask it to help you with your test.

1. (4 points each) Simplify the following:

(a) $\frac{5(3x^2+1)^4(6x)(2x^3-1)^4-4(2x^3-1)^3(6x^2)(3x^2+1)^5}{(2x^3-1)^8}$ $= \frac{6x(3x^2+1)^4\cancel{(2x^3-1)^3}[5(2x^3-1)-4x(3x^2+1)]}{(2x^3-1)^{\cancel{8}\,5}}$

$= \frac{6x(3x^2+1)^4[10x^3-5-12x^3-4x]}{(2x^3-1)^5}$

$= \boxed{\frac{-6x(3x^2+1)^4(2x^3+4x+5)}{(2x^3-1)^5}}$

(b) $\ln\sqrt{\frac{3e^x\sqrt{x}}{x^2(x-1)^4}}$ $= \frac{1}{2}\ln\left[\frac{3e^x\sqrt{x}}{x^2(x-1)^4}\right]$

$= \frac{1}{2}(\ln 3 + \ln e^x + \frac{1}{2}\ln x - 2\ln x - 4\ln(x-1))$

$= \boxed{\frac{1}{2}(\ln 3 + x - \frac{3}{2}\ln x - 4\ln(x-1))}$

(c) $e^{\left(\frac{1}{2}\right)\ln y - 5\ln(3x) - 3}$ $= e^{\ln y^{1/2}} \cdot e^{\ln(3x)^{-5}} \cdot e^{-3}$

$= y^{1/2}(3x)^{-5}e^{-3}$

$= \boxed{\frac{y^{1/2}}{e^3(3x)^5}}$

2. (8 points) Find the average rate of change of $f(x) = \frac{x}{x+1}$ on [1,2].

$f_{avg} = \frac{f(2)-f(1)}{2-1}$

$= \frac{\frac{2}{2+1} - \frac{1}{1+1}}{1}$

$= \frac{2}{3} - \frac{1}{2}$

$= \boxed{\frac{1}{6}}$

3. (a) (3 points each) Find and simplify the indicated compositions, given $f(x) = \sqrt{2x^2+4}$ and $g(x) = \sqrt{x^2-4}$, and state the domains of each composite function:

(i) $f \circ g$ $= f(g)$

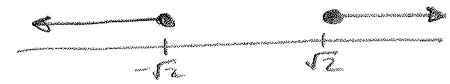
$= \sqrt{2(g)^2 + 4}$

$= \sqrt{2(\sqrt{x^2-4})^2 + 4}$

$= \boxed{\sqrt{2x^2-4}}$

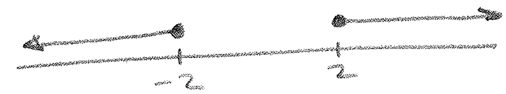
dom$(f \circ g)$: $2x^2 - 4 \geq 0$

$\Rightarrow x^2 \geq 2$



dom(g): $x^2 - 4 \geq 0$

$\Rightarrow x^2 \geq 4$



$\Rightarrow$ Overlap: $(-\infty, -2) \cup (2, \infty)$

Domain of $f \circ g$:

$\boxed{(-\infty, -2) \cup (2, \infty)}$

(ii) $g \circ f$ $= g(f)$

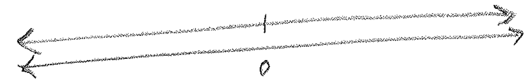
$= \sqrt{(f)^2 - 4}$

$= \sqrt{(\sqrt{2x^2+4})^2 - 4}$

$= \sqrt{2x^2}$

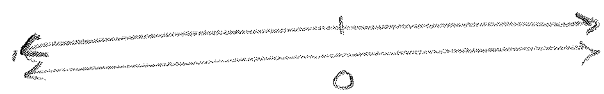
dom$(g \circ f)$: $2x^2 \geq 0$

↳ Always true!



dom(f): $2x^2 + 4 \geq 0$

↳ Always true!

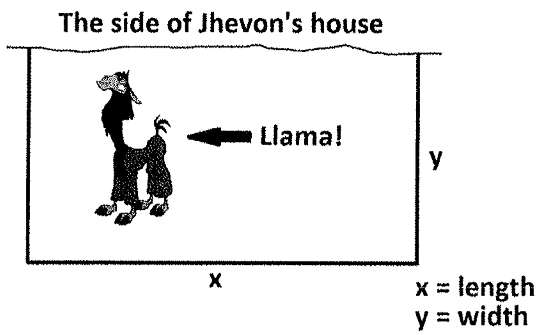


$\Rightarrow$ Overlap: $(-\infty, \infty)$

Domain of $g \circ f$:

$\boxed{(-\infty, \infty)}$

4. (8 points) Jhevon's llamas are out of control, and he decides to build a rectangular fence enclosure in his backyard to keep them out of trouble. He decides to use 100 feet of fencing to make three sides of the fence, with one side of his house forming the fourth side (because who has time to make a four sided enclosure these days anyway?). Figure out **what dimensions the fence must have** so that his llamas have the most room to frolic. Hint: Use the given diagram. Describe the area as a function of the length of one of the sides, and then find the value so that this area function is as large as possible. Use this to figure out the dimensions.



We know: length of fence $= 100$

$\Rightarrow x + 2y = 100$

$\Rightarrow x = 100 - 2y$

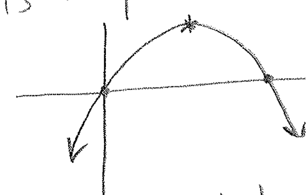
Most room $\Rightarrow$ largest area, So let's look at Area:

$A = xy$

$\Rightarrow A = (100 - 2y)y$

$\Rightarrow A = 100y - 2y^2$

This is a parabola:



Max occurs at the vertex!

$\Rightarrow y = \frac{-100}{2(-2)} = 25\text{ ft}$

$\Rightarrow x = 100 - 2(25) = 50\text{ ft.}$

$\Rightarrow$ $\boxed{\text{Dimensions: } x = 50\text{ft},\ y = 25\text{ ft}}$

5. (10 points each) Solve the following equations:

(i) $e^{x^2-3} - 7 = 0$

$\Rightarrow e^{x^2-3} = 7$

$\Rightarrow x^2 - 3 = \ln 7$

$\Rightarrow \boxed{x = \pm\sqrt{3 + \ln 7}}$

(ii) $\ln(x+1)^2 = -4$

$\Rightarrow (x+1)^2 = e^{-4}$

$\Rightarrow x + 1 = \pm\sqrt{e^{-4}}$

$\Rightarrow \boxed{x = -1 \pm \sqrt{e^{-4}}}$

OR

$\boxed{x = -1 \pm e^{-2}}$

6. (a) (15 points) Let $f(x) = 2 - \frac{5}{x}$. Use the limit definition of the derivative to find $f'(x)$. **No credit will be given for any other method!**

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{2 - \frac{5}{x+h} - \left(2 - \frac{5}{x}\right)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{2} - \frac{5}{x+h} - \cancel{2} + \frac{5}{x}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{5}{x} - \frac{5}{x+h}}{h} \cdot \frac{x(x+h)}{x(x+h)}$$

$$= \lim_{h \to 0} \frac{5(\cancel{x}+h) - 5\cancel{x}}{hx(x+h)}$$

$$= \lim_{h \to 0} \frac{5}{x(x+h)}$$

$$= \boxed{\frac{5}{x^2}}$$

(b) (5 points) Using your answer to part (a), compute the equation of the tangent line to $f(x)$ at the point where $x = 1$. **Write your line in $y = mx + b$ form.**

$x_1 = 1$

$\Rightarrow y_1 = 2 - \frac{5}{1} = -3$

$\Rightarrow (x_1, y_1) = (1, -3)$

$m = f'(1) = \frac{5}{(1)^2} = 5$

Using $y - y_1 = m(x - x_1)$,

we get $y + 3 = 5(x - 1)$

$\Rightarrow \boxed{y = 5x - 8}$

7. (5 points each) Compute the following limits:

(i) $\lim_{x\to -1} \frac{x^2+4x+3}{x^2-x-2} = \lim_{x\to -1} \frac{\cancel{(x+1)}(x+3)}{\cancel{(x+1)}(x-2)}$

$= \lim_{x\to -1} \frac{x+3}{x-2}$

$= \frac{-1+3}{-1-2}$

$= \boxed{-\frac{2}{3}}$

(ii) $\lim_{x\to 0^+} \frac{3x^2+3}{5x^2-4} = \frac{3(0)^2+3}{5(0)^2-4}$

$= \boxed{-\frac{3}{4}}$

(iii) $\lim_{x\to 2^+} \frac{x^2+4x+3}{x^2-x-2}$

$= \lim_{x\to 2^+} \frac{x+3}{x-2}$

$= \frac{5}{0^+}$

$= \boxed{+\infty}$

(ii) $\lim_{x\to\infty} \frac{3x^2+3}{5x^2-4} = \boxed{\frac{3}{5}}$

Ratio of leading coefficients.

Bonus Problems (these will only be counted if all other problems are attempted):

1. (2 points each) State the following rules precisely:

(a) The power rule for derivatives: $\frac{d}{dx} x^n = n x^{n-1}$

(b) The chain rule: $\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$

(c) The quotient rule: $\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{f'g - fg'}{g^2}$

(d) The product rule: $\frac{d}{dx}(f \cdot g) = f' \cdot g + f \cdot g'$

(f) The rule to differentiate a general exponential with base a and power u: $\frac{d}{dx} a^u = u' a^u \ln a$

(g) The rule to differentiate the natural logarithm of a function u: $\frac{d}{dx} \ln u = \frac{u'}{u}$

2. Find $\frac{dy}{dx} = y'$ for the following. Simplify your answers. (3 points each)

(a) $y = \frac{5x^3+3x^2}{2x}$

$= \frac{5}{2}x^2 + \frac{3}{2}x$

$\Rightarrow \boxed{y' = 5x + \frac{3}{2}}$

(b) $y = 2\sqrt{x} + \frac{5}{\sqrt[3]{x}} - \ln(x^2+1)^3$

$= 2x^{1/2} + 5x^{-1/3} - 3\ln(x^2+1)$

$\Rightarrow \boxed{y' = x^{-1/2} - \frac{5}{3}x^{-4/3} - \frac{6x}{x^2+1}}$

(c) $y = \frac{x^6}{4+x^6}$

$\Rightarrow y' = \frac{(4+x^6)(6x^5) - x^6(6x^5)}{(4+x^6)^2}$

$= \frac{6x^5(4+x^6-x^6)}{(4+x^6)^2}$

$\boxed{y' = \frac{24x^5}{(4+x^6)^2}}$

(d) $y = e^{x^2} + x^{x^2}$ $= e^{x^2} + e^{\ln x^{x^2}}$

$= e^{x^2} + e^{x^2 \ln x}$

$= 2xe^{x^2} + \left(2x\ln x + x^2 \cdot \frac{1}{x}\right)e^{x^2 \ln x}$

$\boxed{y' = 2xe^{x^2} + x^{x^2}(2x\ln x + x)}$